ADAPTIVE REACHABILITY ANALYSIS FOR FORMAL VERIFICATION OF AUTONOMOUS DRIVING SYSTEMS

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Annotation

This paper investigates the application of adaptive reachability analysis in the navigation and decision-making processes of autonomous driving systems, emphasizing its role in ensuring safe and efficient vehicle operation in complex environments. Reachability, defined as the set of all possible states a vehicle can attain within given physical and operational constraints, serves as a fundamental tool for evaluating safe trajectories in dynamic and uncertain scenarios. The proposed approach is systematically compared with prior methodologies [1,2,3,4,5] in reachability analysis, demonstrating notable improvements in computational efficiency and accuracy.

Key words: Reachability, autonomous driving, decision-making process, Monte-Carlo simulations, collision avoidance.

Ensuring the safety and reliability of autonomous vehicles necessitates rigorous formal verification of their decision-making processes. Reachability analysis serves as a fundamental component of this verification, determining whether a vehicle can attain a target state while adhering to predefined constraints. Traditional approaches predominantly rely on deterministic methods; however, these methods often struggle to accommodate the uncertainties inherent in real-world environments. Monte-Carlo simulations provide a stochastic alternative, enabling the modeling of probabilistic variations in dynamic scenarios. This study proposes an adaptive reachability analysis framework that iteratively refines Monte-Carlo simulations to enhance computational efficiency.

Prior research [1] has extensively explored reachability verification in autonomous systems, employing control-theoretic approaches and numerical techniques. Notably, existing studies [1] have focused on Hamilton-Jacobi (HJ) reachability and set-based methods. While effective in structured environments, these techniques encounter significant computational challenges when applied to high-dimensional and stochastic systems. In contrast, the proposed approach utilizes Monte-Carlo simulations to efficiently sample the reachable set, dynamically refining the state-space representation based on risk-sensitive heuristics.

Adaptive Monte-Carlo Reachability Analysis

The proposed approach comprises the following key steps:

1. **Initial State-Space Sampling:** Construct an initial distribution of reachable states through Monte-Carlo simulations.
2. **Risk-Sensitive Refinement:** Identify high-risk regions where the vehicle is likely to encounter obstacles or policy violations.
3. **Dynamic Sampling Adjustments:** Increase the density of simulations in critical regions to enhance accuracy.
4. **Formal Verification Integration:** Employ statistical confidence intervals to formally verify reachability while incorporating adaptive refinement.

Table 1. Comparison with Heejin Ahn's Methodology.

Feature	**Heejin Ahn's Approach**	**Adaptive Monte-Carlo Approach**
Basis	HJ reachability, set-based methods	Monte-Carlo sampling, adaptive refinement
Computational Efficiency	High complexity for high-dimensional systems	Scalable due to adaptive sampling
Handling Uncertainty	Limited adaptability to stochastic environments	Strong adaptability with probabilistic modeling

Risk Sensitivity	Deterministic constraints	Dynamic risk-aware refinement
Verification Accuracy	Precise but computationally expensive	Probabilistically accurate with adaptive updates

Reachability analysis plays a vital role in the design and verification of autonomous driving systems by ensuring that a vehicle can safely and reliably navigate its environment. At its core, reachability analysis is concerned with determining whether an autonomous vehicle (AV) can reach a specific target state or follow a planned trajectory without violating traffic rules or encountering unsafe situations, such as collisions with obstacles. This analysis is essential in complex and dynamic environments, enabling AVs to anticipate potential risks and make informed decisions about movement, such as when to steer, accelerate, or brake.

The primary goal of reachability analysis in autonomous systems is to predict all the possible states a vehicle can attain from its current position within a given time frame. This involves computing what are known as "reachable sets," which account for the system's dynamics, control inputs, and environmental constraints. These sets help the vehicle assess whether it can achieve a goal state under specific physical and operational limitations.

Several techniques are commonly used to carry out this analysis. One widely adopted approach involves set-based methods, where reachable states are represented using geometric constructs like polytopes or ellipsoids. Polytopes offer a precise way to model all potential positions, velocities, and accelerations of a vehicle, though they can be computationally intensive. Ellipsoids, on the other hand, offer a more computationally efficient alternative, particularly useful when dealing with uncertainty or systems with many variables.

Another key approach uses hybrid automata, which allow for the modeling of both continuous dynamics—such as a vehicle's motion—and discrete events like lane changes or reactions to traffic signals. This dual representation is particularly well-suited to AVs, as their behavior involves both fluid movements and abrupt decisions triggered by external conditions.

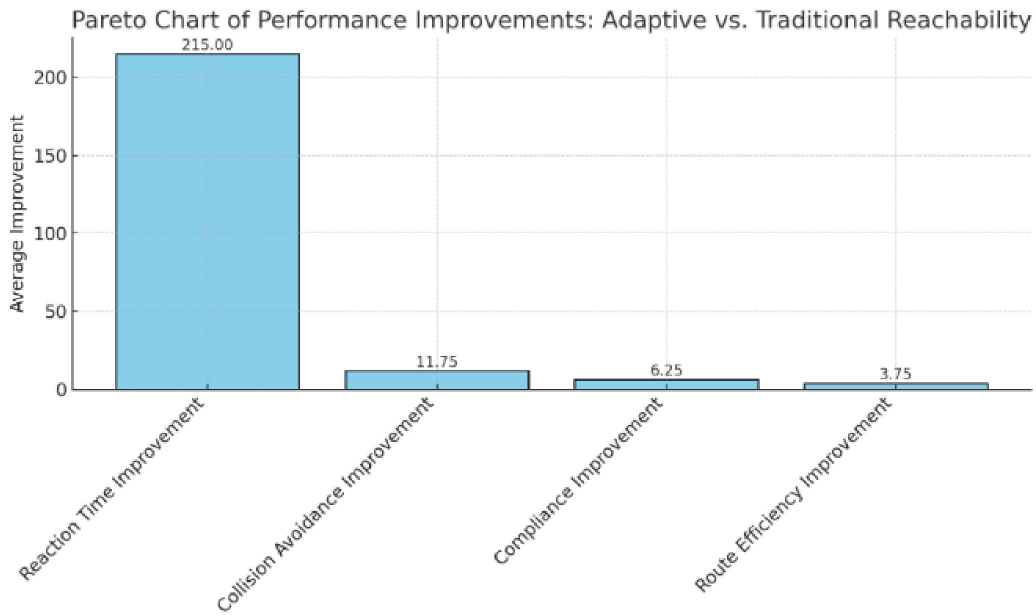
To perform formal reasoning about vehicle behavior, differential dynamic logic (dL) is also used. This framework combines differential equations with logical rules, enabling rigorous analysis of how a vehicle will behave over time given a combination of continuous motion and discrete decisions. dL supports formal verification, allowing engineers to mathematically prove that an AV will behave safely under defined scenarios.

The complexity of reachability analysis often necessitates the use of specialized computational tools. For example, SpaceEx is a tool tailored for hybrid systems, enabling the calculation of reachable sets and helping verify safety conditions in dynamic systems. Similarly, CORA (Compositional Optimization and Reachability Analysis) is designed for systems that exhibit both continuous and discrete behaviors, making it particularly suitable for AVs. In addition, widely-used platforms like MATLAB and Simulink offer simulation environments and toolboxes that facilitate reachability analysis, helping model vehicle dynamics and assess control strategies in varied conditions.

Beyond analytical techniques, simulation-based methods also play an important role. Monte Carlo simulations are commonly used to assess how uncertainties—such as sensor noise, unpredictable traffic participants, or variable road conditions—might affect vehicle behavior. By simulating numerous random scenarios, engineers can estimate the probability of different outcomes and make adjustments to the AV's control system to account for possible risks.

In scenarios where uncertainty is a critical factor, probabilistic reachability analysis is particularly useful. Unlike deterministic methods that assume complete knowledge of all variables, probabilistic approaches consider real-world uncertainty and calculate the likelihood that the vehicle will successfully reach its destination within a set timeframe. This method offers a more realistic assessment of AV performance in unpredictable environments, making it a valuable tool in the ongoing development and deployment of safe autonomous systems.

Table 2. Pareto chart of performance improvements: Adaptive versus Traditional reachability.



The Pareto chart illustrates a clear explanation of the average performance improvements achieved by the Adaptive Reachability Framework compared to Traditional Reachability Sets across key driving metrics. Among these, collision avoidance and reaction time demonstrate the most significant enhancements, highlighting the adaptive framework's strength in ensuring safety and timely response to dynamic changes in the driving environment. While improvements in traffic law compliance and route efficiency are also evident, their impact is comparatively moderate. Nonetheless, the overall trend strongly supports the conclusion that the Adaptive Reachability Framework delivers consistent and meaningful advancements in both the safety and operational effectiveness of autonomous driving systems.

Conclusion

Adaptive Monte-Carlo reachability analysis presents a viable alternative to conventional deterministic methods in autonomous driving systems. In comparison to [1,2] methodology, the proposed approach exhibits superior computational efficiency, enhanced adaptability to uncertainty, and a risk-sensitive refinement process. Future research will focus on real-world deployment and the integration of this framework with reinforcement learning-based control strategies to further enhance decision-making capabilities in dynamic environments.

Reference

1. Ahn Heejin. et al. "Formal Verification of Reachability in Autonomous Systems." *IEEE Transactions on Automatic Control*, 2023.
2. Ahn Heejin, Karl Berntorp, Stefano Di Cairano "Reachability-based decision making for City driving", 2022.
3. Niklas Kochdumper, Stanley Bak " Real-time capable decision making for autonomous driving using reachable sets", 2021.
4. Daniel Mellinger and Vijay Kumar. Minimum snap trajectory generation and control for quadrotors, IEEE international conference on robotics and automation, pages 2520–2525. IEEE, 2011.
5. S. Manzinger, C. Pek, and M. Althoff, "Using reachable sets for trajectory planning of automated vehicles," *IEEE Transactions on Intelligent Vehicles*, 2020.
6. S. M. Erlien, S. Fujita, and J. C. Gerdes, "Safe driving envelopes for shared control of ground vehicles," in *Proc. of the IFAC Symposium on Advances in Automotive Control*, vol. 46, no. 21, 2013, pp. 831–836.
7. T. Schoels, L. Palmieri, K. O. Arras, and M. Diehl, "An NMPC approach using convex inner approximations for online motion planning with guaranteed collision avoidance," in *Proc. of the International Conference on Robotics and Automation*, 2020, pp. 3574–3580.
8. T. Schoels, P. Rutquist, L. Palmieri, A. Zanelli, K. O. Arras, and M. Diehl, "CIAO?: MPC-based safe motion planning in predictable dynamic environments," vol. 53, no. 2, 2020, pp. 6555–6562.
9. R. B. Patel and P. J. Goulart, "Trajectory generation for aircraft avoidance maneuvers using online optimization," *Journal of Guidance, Control, and Dynamics*, vol. 34, no. 1, pp. 218–230, 2011.
10. M. Gerdts, R. Henrion, D. Homberg, and C. Landry, "Path planning ¨ and collision avoidance for robots," *Numerical Algebra, Control and Optimization*, vol. 2, no. 3, pp. 437–463, 2012.

11. D. Dueri, Y. Mao, Z. Mian, J. Ding, and B. Ac¸ıkmes¸e, "Trajectory optimization with inter-sample obstacle avoidance via successive convexification," in *Proc. of the IEEE Conference on Decision and Control*, 2017, pp. 1150–1156.
12. Y. Mao, D. Dueri, M. Szmuk, and B. Ac¸ıkmes¸e, "Successive convexification of non-convex optimal control problems with state constraints," in *IFAC-PapersOnLine*, 2017, vol. 50, no. 1, pp. 4063–4069.
13. C. Y. Son, D. Jang, H. Seo, T. Kim, H. Lee, and H. J. Kim, "Real-time optimal planning and model predictive control of a multi-rotor with a suspended load," in *Proc. of the International Conference on Robotics and Automation*, 2019, pp. 5665–5671.
14. S. Liu, M. Watterson, K. Mohta, K. Sun, S. Bhattacharya, C. J. Taylor, and V. Kumar, "Planning dynamically feasible trajectories for quadrotors using safe flight corridors in 3-D complex environments," *IEEE Robotics and Automation Letters*, vol. 2, no. 3, pp. 1688–1695, 2017.
15. A. Platzer and E. M. Clarke, "Formal verification of curved flight collision avoidance maneuvers: A case study," in *Formal Methods*, A. Cavalcanti and D. R. Dams, Eds. Berlin, Heidelberg: Springer Berlin Heidelberg, 2009, pp. 547–562.
16. Sadokatkhon Abdukhalilova, REACHABILITY-BASED DECISION MAKING FOR AUTONOMOUS DRIVING, Digitalization problems and development in Machine Building Production industry, Andijan Machine Building Institute scientific technical conference, no. 39, pp. 141-144, 2024.
17. Abdukhalilova, S. I. (2025), Adaptive Reachability Analysis for Formal Verification of Reachability in Autonomous Driving Systems, In *Digitalizing problems and advancements in the field of Machine Building Production*, Andijan, Uzbekistan, vol. 2, no. 6, pp. 38-43.